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TENSILE STRESSES IN TARGET UPON COLLISION OF RIGID BODIES

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Introduction. When an explosion occurs in a rigid body, almost half the energy of the explosives passes into the shock wave [1], and therefore shock wave processes play an important part in the breaking up of rocks and metals by impulsive loading. From the physical point of view the process of brittle failure is characterized by the detachment of the material on the free surface under the effect of the tensile stresses in the rock massif or metal plate in case of wave interference [2, 3].

The spalling of a layer of material is investigated in order to reveal its mechanism and the laws governing the process of failure in material under intensive loads ([14, 15] etc.) and to use it in practical problems, e.g., in breaking up rocks [11, 16-18] or for protection at high-speed collision of rigid bodies [19].

In early investigations, in analogy with static loading, the strength upon intensive loading was described by the critical normal breaking stresses σ_*, characteristic for the given material at the stipulated loading conditions [4, 20], and the criterion of failure was adopted in the form

$$\sigma = \sigma_*. \quad (0.1)$$

When a normal tensile stress at any point of the body attained the value σ_*, it was considered the condition leading to failure of the material. This criterion has not lost its importance up to the present [3, 5, 11, 21] because it is a necessary, albeit not sufficient, condition of failure. It was already noted in [20] that spalling is also affected by the shape of the stress wave, and it was assumed that σ_* depends on the conditions of loading and on the stress distribution in the body. Experimental measurements of the resistance to detachment upon spalling, conducted on the same materials by different authors, yielded widely

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differing results [10, 12, 22]: by one order of magnitude in [5] and [23], [24] and [11], for aluminum values between 9.6 and 58 were obtained [25-27], for copper from 69 to 128 [5], and for mild steel from 70 to 105 kbar [24]. These differences become comprehensible [10, 28] if we accept the kinetic concept of the strength of rigid bodies [29, 30], in accordance with which strength is not a boundary value having the character of some constant, and failure due to spalling depends not only on the amplitude of the tensile stresses but also on time factors. The process of failure is characterized mainly by the endurance of the specimen, i.e., the time τ from the instant of applying the load up to failure, at constant stress σ and temperature T:

$$\tau = \tau_0 \exp\left(\frac{U_0 - \gamma\sigma}{kT}\right), \qquad (0.2)$$

where k is the Boltzmann constant; τ_0 is the period of the normal thermal vibrations of the atoms in the body, their order of magnitude being 10^{-13} sec; U_0 is the initial energy barrier, which coincides well with the sublimation temperature of the body; γ is a structure- and sensitivity-dependent value given by the state of the body. In [6] other empirical criteria [31-34] are also given; they are used for describing plastics and glasses. In several works [9, 24, 34, 35] the dependence of the strength on the deformation rate, the pressure gradient, or on the rate of pressure change in a compressional wave was experimentally determined, and this is also unambiguously correlated to the duration of the tensile stresses; in [5] a time criterion for metals upon compression of a plate by a triangular impulse was experimentally obtained, and this, as was shown in [12], reduces to the criterion (0.2).

However, it was found experimentally [8, 9, 13, 15, 22, 28, 36] that Zhurkov's equation for endurance does not apply in the loading time under consideration, viz., 10^{-7}-10^{6} sec. And though in [15] a single monotonic curve for the dependence of strength on time was experimentally obtained within the range of changes of endurance from 10^{-7} to 10^{6} sec for the aluminum alloy V95, it is obvious that the nature of the time dependence of failure under intensive loading is quite different [37].

Other time criteria were submitted in [11, 14, 38]. The criterion

$$\int_0^\tau \sigma(t)\, dt = J_*,$$

introduced in [11], indicates that failure may occur at the instant τ, when the impulse of the tensile stresses in the cross section of the plate attains some experimentally determined critical value J_*. The time criterion [14] has the energetic meaning:

$$\frac{1}{AE} \int_0^{l_0} \sigma^2 dx = \varepsilon_*, \qquad (0.3)$$

where $A = 2(1 - \mu)/(1 + \mu)(1 - 2\mu)$, μ is Poisson's constant, E is the modulus of elasticity, ε_* is the specific work of detaching material per unit surface, l_0 is the length of the tensional wave; this means that failure occurs when the specific elastic energy contained in the tensional wave attains ε_*. If the tensional wave is constant in time, (0.3) coincides with the criterion of failure obtained in [38]:

$$\frac{c}{AE} \int_0^{t_0} \sigma^2 dt = \varepsilon_*,$$

where $t_0 = l_0/c$, being the velocity of the sound in the material.

To find the quantitative criteria of spall, it is essential to know the amplitude and duration of the tensile stresses; but since most works use methods connected with the measurement of the translational velocity of the free surface of the body [10, 15, 39, 40] and the sought values are calculated from experimentally obtained values, establishing the de-

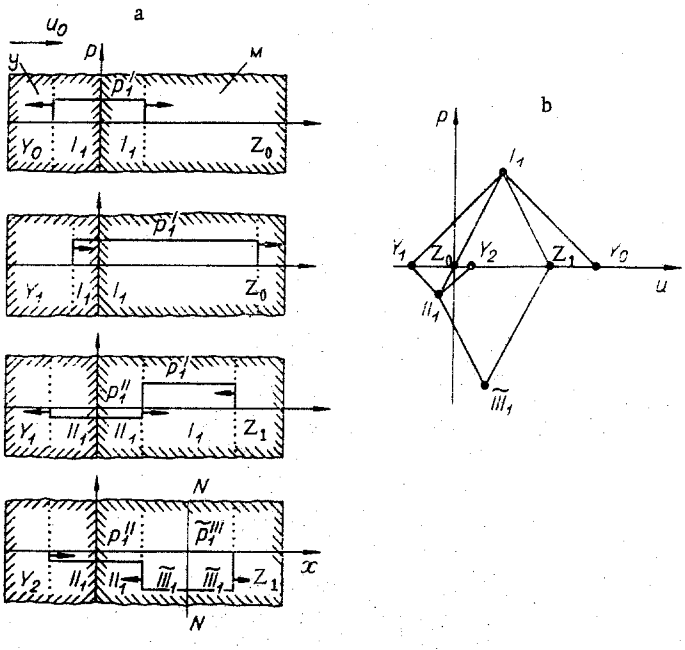

Fig. 1

pendence of the stress amplitude on the velocity of the free surface (or the impact velocity) is a serious problem.

Most authors calculate the tensile stresses in the tensional wave, if it is rectangular, using the acoustic approximation

$$\sigma = (1/2)\rho u_* c, \tag{0.4}$$

where ρ is the density of the material of the plate; u_* is the threshold velocity of impact at which spall occurs [4, 13, 14, 19, 28]. However (0.4) applies only to impact of two plates of the same material because in impact of unequal plates the impulse of compression has a stepped shape [41].

It is quite justified to use the acoustic approximation for shock waves in a rigid body with pressure amplitudes of tens or hundreds of kilobars [42]. Such shock waves are "weak," they differ little from acoustic ones and propagate with a velocity close to the velocity of sound. On the other hand, they cannot be considered very weak, and therefore the effects associated with the strength of solids may be neglected. The stress tensor in bodies compressed by a shock wave is assumed to be spherical, like in an ideal gas or liquid, because compared with ultimate strength or critical shear stress, the pressure at the front of the shock wave is large. Here the velocity of sound is determined by the compressibility of the substance, like in a gas or liquid.

It was shown in [41] that for weak shock waves the equations determining the Hugoniot line of the substance in the p, u diagrams are linear, and this permits the parameters of the shock waves to be determined in the simplest and most economical way [43].

The present article proposes a method of calculating the tensile stresses during the collision of a plate with a two-layer plate and the methods of controlling the stress amplitudes in the two-layer plate. For the sake of simplicity of the analysis it is assumed that failure occurs when the strength criterion (0.1) is attained.

1. Let us examine the case of collision of two plates of different materials. The maximum tensile stress in section N of the target has the form [43]

$$\sigma = \tilde{p}_1^{\rm III} = -\frac{2R_{\rm y}R_{\rm Z}^2}{(R_{\rm y}+R_{\rm Z})^2}u_0, \tag{1.1}$$

TABLE 1

Punch material \ Pad material	Tungsten	Copper	Dur-alumin	Poly-ethylene
Target material tungsten				
Tungsten	−261	−132	−39	−0,8
Copper	−219	−157	−63	−1,7
Duralumin	−131	−128	−77	−3,8
Polyethylene	−20	−27	−29	−10,2
Target material copper				
Tungsten	−157	−94	−32	−0,8
Copper	−132	−112	−53	−1,7
Duralumin	−79	−92	−64	−3,7
Polyethylene	−12	−19	−24	−10,0
Target material duralumin				
Tungsten	−77	−54	−23	−0,7
Copper	−65	−64	−37	−1,6
Duralumin	−39	−53	−45	−3,5
Polyethylene	−6	−11	−17	−9,4

Note. If the material of the pad is the same as that of the target, it is identical to the case with no pad.

where $R_i = \rho_i c_i$ ($i = y, z$), R_i being the acoustic stiffness of the material; the subscript y refers to the punch moving at velocity u_0; the subscript z refers to the target. Figure 1a shows the successive patterns of the propagation of the shock waves in the punch y and the target z. The states in the plates are designed Y_0 for the punch and Z_0 for the target before impact, I after impact, Y_1 and Z_1 after reflection of the waves from the free surfaces of the punch and the target, II_1 after interaction of the wave with the contact boundary of the plate, $\widetilde{III}_1$ after interaction of the waves in the section N of the target. Here we also give the pressure profiles for the case $R_y < R_z$. The process of collision is considered unidimensional, the pattern of propagation of the shock waves is considered acoustic [43]. It is also assumed that $t_y < t_z$, where $t_i = l_i/c_i$ ($i = y, z$), t_i is the time of propagation of the shock wave in the plate between its two boundaries; l_i is the thickness of the plate. The amplitude of the tensile stresses σ (1.1) is found by the method of the p, u diagrams. The state in the target brought about by the interaction of the two waves (the wave reflected from the free surface of the target and the wave reflected from the free surface of the punch and passing through the punch–target interface) is indicated in the p, u diagram (Fig. 1b) by point $\widetilde{III}_1$ whose coordinates $\tilde{u}_1^{III}$ and $\tilde{p}_1^{III}$ indicate the mass velocity and the pressure in the tensional wave. The rest of the notation in the p, u diagram has been explained above.

The function $\tilde{p}_1^{III}(R_y, R_z)$ (1.1) is determined for $R_y > 0$, $R_z > 0$. In this region of the arguments it is continuous and can assume only values $\tilde{p}_1^{III} < 0$. With fixed acoustic stiffness of the target $R_z = \text{const}$, the minimum of this function is attained at $R_y = R_z$, and it is

$$\tilde{p}_1^{III}(R_y)_{R_y=R_z} = -\frac{R_z}{2} u_0. \quad (1.2)$$

If R_y is fixed, then the function $\tilde{p}_1^{III}(R_z)$, with R_z increasing from zero to infinity, decreases from zero and approaches $-2R_y u_0$ asymptotically.

We shall investigate two problems: a) by selecting the material of the punch to ensure spall of the target; b) by selecting the material of the target to ensure its integrity.

Since it is a necessary condition that the amplitude of the tensile stresses attain the magnitude of the normal critical breaking stresses (under the given loading conditions), in solving the first problem the following relationship must be maintained:

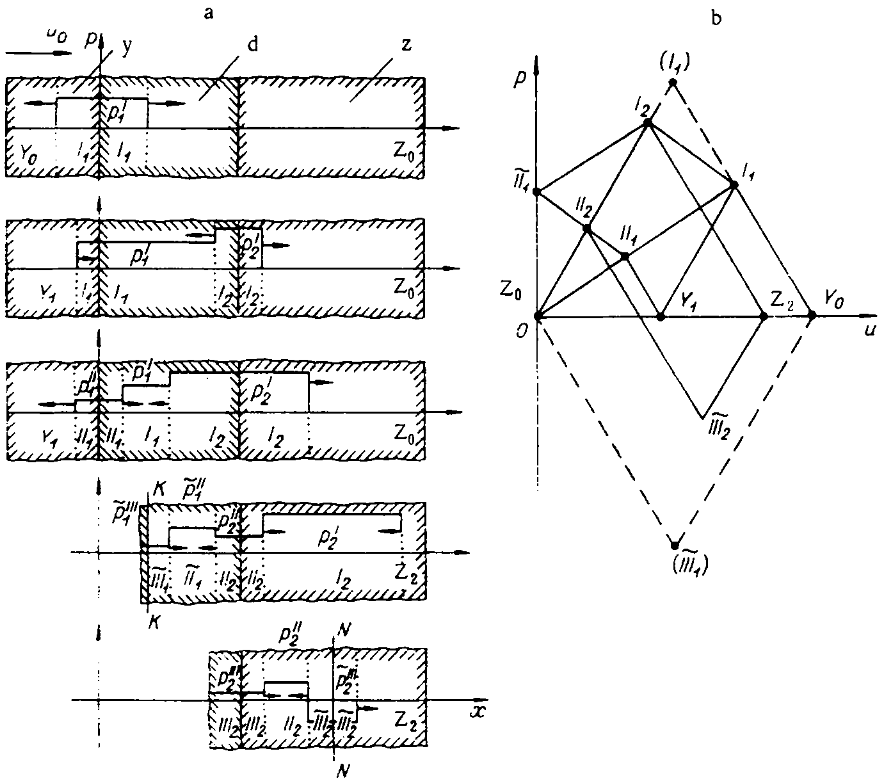

Fig. 2

$$|\tilde{p}_1^{III}| = \frac{2R_y R_Z^2}{(R_y + R_Z)^2} u_0 \geqslant |(\sigma_Z)_*|.$$

In view of (1.2) the maximum value of $|\tilde{p}_1^{III}|$ can be obtained when $R_y = R_Z$. It is therefore advisable, if the target is to fail in section N, to make the punch of the same material as the target.

Taking this into account, the condition of dynamic failure of the target material assumes the form

$$(R_y/2)u_0 \geqslant |(\sigma_Z)_*|.$$

Table 1 gives some theoretical values of $\tilde{p}_1^{III}$(kbar) in the section N for different materials for the punch and the target at $u_0 = 500$ m/sec. The values of ρ and c for the plate materials are taken from [44].

Since the duration of the tensile stresses with the maximum amplitudes in the section is determined by the thickness l_y of the punch,

$$t = 2l_y/c_y,$$

it is expedient to increase the thickness of the punch while retaining the rest of its parameters, as far as this is possible with a view to design considerations and the attenuation of the amplitude.

In solving the second problem, the following condition must be fulfilled:

$$|\tilde{p}_1^{III}| = \frac{2R_y R_Z^2}{(R_y + R_Z)^2} u_0 < |(\sigma_Z)_*|. \tag{1.3}$$

For that R_Z must be reduced. However, since it is always possible to select $R_y = R_Z$, the material of the target must be chosen such that relations (1.2) and (1.3) are both maintained we obtain

$$(R_z/2)u_0 < |(\sigma_z)_*|.$$

2. New possibilities in the expedient control of the magnitude of the tensile stresses in the target are provided by the introduction of a plate between the target and the punch, i.e., by covering the target with a pad (Fig. 2a).

The shock waves generated by the collision of the punch with the pad, having the parameters

$$p_1^I = \frac{R_y R_d}{R_y + R_d} u_0, \quad u_1^I = \frac{R_y}{R_y + R_d} u_0, \tag{2.1}$$

reach the opposite boundaries of the colliding plates ($R_d = \rho_d c_d$ is the acoustic stiffness of the pad material). One of the shock waves, propagating to the right through the pad, reaches the interface with the target, and as a result of the discontinued failure it induces a new state in these plates (point I_2 in the p, u diagram shown in Fig. 2b which is plotted for the case $R_y = R_z > R_d$). It is determined from the system of equations

$$p_2^I = R_z u_2^I, \quad p_2^I = -R_d(u_2^I - u_1^I).$$

When we solve it, we obtain

$$p_2^I = \frac{2R_y R_d R_z}{(R_y + R_d)(R_d + R_z)} u_0, \qquad u_2^I = \frac{2R_y R_d}{(R_y + R_d)(R_d + R_z)} u_0. \tag{2.2}$$

The wave (2.2) then reaches the free surface of the target, and thereupon the unloading wave begins to propagate through the target toward the left (point Z_2 in the p, u diagram):

$$p_2^z = 0, \quad u_2^z = \frac{4R_y R_z}{(R_y + R_d)(R_d + R_z)} u_0. \tag{2.3}$$

The shock wave (2.1), propagating through the punch toward the left, reaches its free surface and induces an unloading wave which, upon reaching the interface with the pad, induces, in consequence of a renewed discontinuity in failure, waves with parameters [43] (point II_1 in the p, u diagram)

$$p_1^{II} = \frac{R_y R_d (R_y - R_d)}{(R_y + R_d)^2} u_0, \qquad u_1^{II} = \frac{R_y (R_y - R_d)}{(R_y + R_d)^2} u_0. \tag{2.4}$$

If we assume that $t_y < t_d < t_z$ (where $t_d = l_d/c_d$ is the time of propagation of the wave front in the pad between the boundaries of the plate, l_d is the thickness of the pad), then the wave (2.4) encounters in some section $\tilde{K}$ of the pad the wave (2.2) and induces a state characterized by the coordinates of point $\tilde{II}_1$ in the p, u diagram. It is determined from the system of equations

$$\tilde{p}_1^{II} - p_2^I = R_d(\tilde{u}_1^{II} - u_2^I), \quad \tilde{p}_1^{II} - p_1^{II} = -R_d(\tilde{u}_1^{II} - u_1^{II}),$$

and when we solve it, we obtain

$$\tilde{p}_1^{II} = \frac{2R_yR_d(R_yR_z - R_d^2)}{(R_y + R_d)^2(R_d + R_z)}u_0,$$
$$\tilde{u}_1^{II} = \frac{2R_yR_d(R_y - R_z)}{(R_y + R_d)^2(R_d + R_z)}u_0. \tag{2.5}$$

The wave (2.5), propagating through the pad to the right, reaches the interface with the target, and a state characterized by point II_2 in the p, u diagram is induced in both plates. From the system of equations

$$p_2^{II} = R_z u_2^{II}, \quad p_2^{II} - p_1^{II} = -R_d(u_2^{II} - u_1^{II})$$

we obtain the parameters of this state

$$p_2^{II} = \frac{2R_yR_dR_z(R_y - R_d)}{(R_y + R_d)^2(R_d + R_z)}u_0,$$
$$u_2^{II} = \frac{2R_yR_d(R_y - R_d)}{(R_y + R_d)^2(R_d + R_z)}u_0. \tag{2.6}$$

The encounter of wave (2.6) with the unloading wave (2.3) and their interaction induce in some section N a state characterized in the p, u diagram by the coordinates of point $\widetilde{III}_2$ (cf. Fig. 2b). This state is determined from the system of equations

$$\tilde{p}_2^{III} = R_z(\tilde{u}_2^{III} - u_2^M), \quad \tilde{p}_2^{III} - p_2^{II} = -R_z(\tilde{u}_2^{III} - u_2^{II}).$$

Solving this system, we find

$$\tilde{p}_2^{III} = -\frac{4R_yR_d^2R_z}{(R_y + R_d)^2(R_d + R_z)}u_0,$$
$$\tilde{u}_2^{III} = -\frac{4R_y^2R_d}{(R_y + R_d)^2(R_d + R_z)}u_0. \tag{2.7}$$

The function $\tilde{p}_2^{III}(R_y, R_d, R_z)$ (2.7) is determined in the range $R_y > 0$, $R_z > 0$, $R_d > 0$. It is continuous and can only be negative. Let us examine the change in the function $\tilde{p}_2^{III}(R_d)$ with R_y = const and R_z = const. Obviously, since at $R_d = 0$ and $R_d \to \infty$ $\tilde{p}_2^{III}(R_d) = 0$, this function has an extremum when

$$R_d = \frac{R_y}{2} + \sqrt{\frac{R_y^2}{4} + 2R_yR_z}. \tag{2.8}$$

It follows from (2.8) that $R_d > R_y$.

If we fix R_z and R_d and examine the change in the function $\tilde{p}_2^{III}(R_y)$, we find that it also tends toward zero when $R_y = 0$ and $R_y \to \infty$, and it consequently has a minimum in the stipulated region of determination. It is attained when $R_y = R_d$ and is equal to

$$\tilde{p}_2^{III}(R_y)_{R_y=R_d} = -\frac{R_dR_z}{R_d + R_z}u_0. \tag{2.9}$$

Furthermore, we put Re = const and R_d = const, and examine the change in the function $\tilde{p}_2^{III}(R_z)$. Its values in the region of the argument to be determined decrease from zero, when $R_z = 0$, to

$$\tilde{p}_2^{III} = -\frac{4R_yR_d^2}{(R_y + R_d)^2}u_0$$

when $R_z \to \infty$.

Thus, this condition has the form

$$0 \geqslant \tilde{p}_2^{III}(R_z) \geqslant -\frac{4R_y R_d^2}{(R_y + R_d)^2} u_0.$$

If failure is to be induced in the target and the material of the punch may be chosen, the tensile stress with maximum amplitude, equal to the value of $\tilde{p}_2^{III}$ from (2.9), can be obtained when $R_y = R_d$. We want to point out that if also $R_z = R_d$, then (2.9) reduces to (1.2). This case is shown in Fig. 2b by the dashed lines. The stresses in section N for different pads are presented in Table 1.

To ensure strength of the target by choosing the material of the target and of the pad, care should be taken that R_d differs considerably from its value obtained by (2.8). Since in (2.8) $R_d > R_y$, it is an advantage to choose $R_d < R_y$. For instance, if the punch and the target are made of copper and the pad of duralumin, the tensile stresses are reduced to —53 kbar, whereas if there were no pad, they would amount to —112 kbar. Impact of a tungsten punch on a copper target induces in the target tensile stresses of —94 kbar; the introduction of a duralumin pad causes tensile stresses of —32 kbar. In any case R_d and R_z have to be chosen such that the relationship

$$|\tilde{p}_2^{III}| = \frac{4R_y R_d^2 R_z}{(R_y + R_d)^2 (R_d + R_z)} u_0 < |(\sigma_z)_*|$$

is maintained.

If we assume that $R_y = R_d$ will always be chosen, R_d and R_z must be chosen such that

$$\frac{R_d R_z}{R_d + R_z} u_0 < |(\sigma_z)_*|.$$

Assume that the target is lined at the rear by a plate with acoustic stiffness $R_o = \rho_o c_o$. It follows from (2.5) that the stresses induced in the target in this case have the form

$$\tilde{p}_1^{II} = \frac{2R_y R_z (R_y R_o - R_z^2)}{(R_y + R_z)^2 (R_z + R_o)} u_0 \qquad (2.10)$$

[in (2.5) we must take R_z instead of R_d, and the acoustic stiffness R_o instead of R_z].

Obviously, tensile stresses are induced when the expression in parentheses in the numerator is negative, i.e., when the unequality

$$R_y / R_z < R_z / R_o$$

holds.

Consequently, by reducing the acoustic stiffness of the target or by increasing the acoustic stiffness of the lining the tensile stresses in the target can be eliminated. However, the lining itself will be subjected to the action of the tensile stresses (2.7) [in (2.7) R_d and R_z are replaced by R_z and R_o, respectively],

$$\tilde{p}_2^{III} = -\frac{4R_y R_z^2 R_o}{(R_y + R_z)^2 (R_z + R_o)} u_0,$$

which may cause its failure.

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SHOCK ADIABATS AND PROFILES OF WEAK SHOCK WAVES IN METALS

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1. Description of the Parameters of the Medium

The mathematical model of an isotropic medium, suggested in [1-3], supposes that the substance is defined by an internal energy E per unit mass of substance and by a characteristic time τ of relaxation of tangential stresses. The internal energy E is related by the equation of state of the medium

$$E = E(\alpha, \beta, \gamma, S) \quad (1.1)$$

with the density of the entropy S and the quantities α, β, γ. The parameters α, β, and γ are the logarithms of the "elastic" extensions k_1, k_2, and k_3 along the principal axes of elastic deformation

$$\alpha = \ln k_1,\ \beta = \ln k_2,\ \gamma = \ln k_3.$$

The equations of state of the type (1.1) are given in [2] for iron (α-phase), aluminum, copper, nickel, lead, and titanium. The characteristic time τ of relaxation of tangential stresses is determined by the formula for its dependence on the stressed state of the medium

$$\tau = \tau(\sigma, T), \quad (1.2)$$

where T is the temperature; $\sigma = (1/\sqrt{2})\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$ is the intensity of the tangential stresses (σ_1, σ_2, and σ_3 are the principal stresses). The form of this relation is given in [3] for iron, aluminum, copper, and lead.

When calculating shock waves, a relation is used which is a variant of the corresponding formula from [3]:

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